Studies in Chemical Process Design and Synthesis:

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III. A Simple and Practical Approach to the Optimal Synthesis of Heat Exchanger Networks

An algorithmic-evolutionary approach to the systematic synthesis of minimum cost networks of exchangers, heaters, and coolers is proposed. The new approach is easy to apply by hand calculations, requiring no special mathematical background and computational skill from the user. In addition to generating many cheaper networks for sample problems ranging in size from 4 to 10 streams as compared to previous studies, the proposed method provides an explicit theoretical guidance on the optimal exchange among hot and cold streams and on the optimal locations of heating and cooling utilities in the network. It also has a provision for the use of stream splitting and for generating a cyclic network. The new method is particularly effective in the synthesis of minimum cost networks for industrial crude unit preheat recovery.

SCOPE

An important process design problem is the synthesis of minimum cost networks of exchangers, heaters, and/or coolers to transfer the excess energy from a set of hot streams to streams that require heating (cold streams). The general techniques that have been developed recently for solving this problem have included the heuristic approach based on the use of rules of thumb (for example, Masso and Rudd, 1969), algorithmic methods involving often some established optimization principles (for example, Lee et al., 1970), and evolutionary strategies wherein improvements are systematically made to an initially created feasible network (for example, McGalliard and Westerberg, 1972). In some situations, two or more of these techniques have been used together in the synthesis (for example, Menzies and Johnson, 1972). Often, applications of these previous synthesis techniques require some special mathematical background and computational skill from the user. Table 4 summarizes the previous studies on the systematic synthesis of heat exchanger networks.

In the majority of published studies on the synthesis of heat exchanger networks, there has been very little theoretical guidance on how hot and cold process streams are to be optimally exchanged as well as where heaters and coolers should best be placed in the network. For instance, it has always been assumed that if a process stream does not reach its desired output temperature, it reaches its output temperature by an exchange with a utility stream in an auxiliary heater or cooler. The exchange or matching of a process stream with heating or cooling utility is thus restricted to its last matching step. This approach, however, will often exclude many cheaper networks. Furthermore, in most previous synthesis techniques, there has been no provision for the use of stream splitting and for the generation of a cyclic network in which a stream is exchanged more than once with another stream. Both of these latter aspects will frequently lead to the creation of networks with high energy recovery and less total cost. Finally, perhaps the most serious limitations of many previous synthesis techniques are the combinatorial difficulties resulting from the large number of enumeration required to synthesize an optimal network, and the corresponding excessive computation time and computer storage needed. The development of a simple and practical approach to overcome such problems associated with the existing synthesis techniques thus presents a serious challenge.

In this work, a new approach to the systematic synthesis of minimum cost networks of exchangers, heaters, and coolers is presented. The synthesis strategy adopted is to create a network of a minimum investment cost with a practically fixed and minimum utility operating cost, while achieving a maximum amount of heat exchange among hot and cold process streams. Specifically, optimization principles are used first to explore analytically the

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necessary conditions for minimizing the total heat transfer area of the network when the investment costs of exchangers, heaters, and coolers are approximated as linear functions of their heat transfer areas. The necessary conditions derived suggest a simple and practical algorithm called the minimum area algorithm, for the synthesis of a minimum area or a nearly minimum cost network of exchangers, heaters, and coolers. The next step of the proposed approach is to employ a set of simple evolutionary rules to systematically modify the resulting minimum area network so that total cost of investment and utilities can be reduced. Examples taken from previous publications in the literature ranging in size from 4 to 10 streams are used to illustrate the new synthesis method, and a detailed comparison with previous studies is also given.

CONCLUSIONS AND SIGNIFICANCE

The present work demonstrates a simple and practical approach for the systematic synthesis of minimum cost networks of exchangers, heaters, and coolers. It is shown that the application of the proposed minimum area algorithm and evolutionary rules have successfully generated many cheaper networks for sample problems ranging in size from 4 to 10 streams as compared with previous studies. Furthermore, the proposed approach is easy to apply by hand calculations, requiring no special mathematical background and computational skill from the user. While the computer automation of the new method is definitely possible, it is not necessarily needed in solving large scale synthesis problems. Based on the theoretical developments and illustrative examples presented, the following major conclusions and significance of this work can be summarized:

1. The minimum area algorithm provides a simple and practical approach to the synthesis of the whole network of exchangers, heaters, and coolers as an integrated system, with the optimal locations of heaters and coolers also determined. Several explicit rules for determining the maximum amount of heat exchange among process streams as well as for matching process and/or utility streams in a given synthesis problem can be easily visualized and identified on a simple, auxiliary graphical representation called the heat content diagram (Nishida et al., 1971). These rules can be easily applied to generate a nearly minimum cost network in large scale problems. In addition to eliminating the combinatorial difficulties which are commonly associated with other synthesis techniques, the algorithm has an explicit provision for the use of stream splitting and for generating a cyclic network. The nearly minimum cost network obtained by the algorithm also serves as one of the best initial networks for the applications of many other synthesis methods.

2. The proposed evolutionary rules can be easily applied to improve the nearly minimum cost network obtained by the minimum area algorithm and by other synthesis techniques which do not satisfy the theoretical guidance presented in this work. For all the example problems solved in this work, simple applications of the proposed rules have successfully generated many cheaper networks as compared with previous studies. The new method is particularly effective in the the synthesis of minimum cost networks for industrial crude unit preheat recovery.

PROBLEM STATEMENT

The synthesis problem to be considered has been defined by Masso and Rudd (1969). Briefly, there are M hot streams S_{hi} ($i = 1, 2, \ldots, M$) to be cooled and N cold streams S_{cj} ($j = 1, 2, \ldots, N$) to be heated. Associated with each stream are its known input temperature T_i, output temperature T_i^*, and heat capacity flow rate W_i. There are also available auxiliary steam heaters and water coolers S_{uk} ($k = 1, 2, \ldots, p$) called utilities. The problem is to create a minimum cost network of exchangers, heaters, and/or coolers so that the desired output temperature of each process stream is reached. In general, the investment costs for the i^{th} exchanger, heater, and cooler, denoted by C_{Ei}, C_{Hi}, and C_{Ci}, respectively, can be correlated to their corresponding heat transfer areas A_{Ei}, A_{Hi}, and A_{Ci} by the empirical expressions $C_{Ei} = a\,A_{Ei}^b$, $C_{Hi} = a\,A_E^b$, and $C_{Ci} = a\,A_{Ci}^b$. The total cost of investment and utility of the network to be minimized can be expressed as

$$J = \delta\left[\sum_i a\,A_{Ei}^b + \sum_i a\,A_{Hi}^b + \sum_i a\,A_{Ci}^b\right] + \sum_k \sum_l u_k\,S_{ukl} \quad (1)$$

For convenience, the following simplifying assumptions, which have been used in most previous studies of synthesis of heat exchanger networks, are included: the use of countercurrent shell and tube exchangers, no phase changes of process streams, and equal values of the effective heat transfer coefficients for all exchangers. The implications and limitations of these assumptions will be discussed further in this paper. The illustrative examples in this paper are all taken from the literature, and their specifications of process streams and design data are summarized in Tables 1 and 2.

THEORETICAL DEVELOPMENTS

An Algorithmic-Evolutionary Synthesis Strategy

The synthesis task usually begins by either exchanging heat among hot and cold process streams or by heating and cooling process streams through auxiliary heaters and coolers. The desired minimum cost network will depend on the investment cost for the exchangers, heaters, and coolers, as well as the utility operating cost. In general, the utility operating cost represents a significant portion of the total cost of the network, and it is most desirable to exchange as much heat as is technically possible among hot and cold process streams to reduce the utility operating cost. In a given heat exchanger network synthesis problem, the maximum amount of heat exchange among process streams Q_R is usually fixed by the total heat contents of hot (Q_h) cold (Q_c) streams, and by the input and output temperatures of hot and cold streams, as well as by the minimum approach temperature for the exchanger ΔT_M (about 15° ~ 20°F). When the synthesis strategy is to achieve a maximum amount of heat exchange among hot and cold process streams, the heat duty for utilities is

Table 1. Process Streams Specifications

Problem 4SP1 (Lee et al., 1971)

Stream	Capacity flow rate (Btu/hr-°F)	Input temp. (°F)	Output temp. (°F)
S_{c1}	14 450.1	140	320
S_{c2}	11 530.0	240	500
S_{h1}	16 666.8	320	200
S_{h2}	20 000.0	480	280

Problem 4SP2 (Ponton and Donaldson, 1974)

Stream	Capacity flow rate (Btu/hr-°F)	Input temp. (°F)	Output temp. (°F)
S_{c1}	70 000.0	25	420
S_{h1}	20 000.0	500	110
S_{h2}	50 000.0	430	230
S_{h3}	30 000.0	400	110

Problem 5SP1 (Masso and Rudd, 1969)

Stream	Capacity flow rate (Btu/hr-°F)	Input temp. (°F)	Output temp. (°F)
S_{c1}	21 600.0	100	400
S_{c2}	24 500.0	150	360
S_{c3}	24 700.0	200	400
S_{h1}	31 500.0	480	250
S_{h2}	25 200.0	400	150

Problem 6SP1 (Lee et al., 1970)

Stream	Capacity flow rate (Btu/hr-°F)	Input temp. (°F)	Output temp. (°F)
S_{c1}	16 000.0	100	430
S_{c2}	32 760.0	180	350
S_{c3}	26 350.0	200	400
S_{h1}	28 000.0	440	150
S_{h2}	23 800.0	520	300
S_{h3}	33 600.0	390	150

Problem 10SP1 (Pho and Lapidus, 1973)

Stream	Capacity flow rate (Btu/hr-°F)	Input temp. (°F)	Output temp. (°F)
S_{c1}	14 450.0	140	320
S_{c2}	11 530.0	240	431
S_{c3}	16 000.0	100	430
S_{c4}	32 760.0	180	350
S_{c5}	26 350.0	200	400
S_{h1}	16 670.0	320	200
S_{h2}	20 000.0	480	280
S_{h3}	28 000.0	440	150
S_{h4}	23 800.0	520	300
S_{h5}	33 600.0	390	150

practically fixed by the residual heat content, and the corresponding utility operating cost is thus practically at its minimum value. Consequently, the minimum cost network synthesis problem now becomes a problem of creating a network of a minimum investment cost of exchangers, heaters, and/or coolers with a practically fixed and almost minimum utility operating cost, while achieving a maximum amount of heat exchange among hot and cold process streams.

In this paper, a combined algorithmic and evolutionary strategy is developed to create such a minimum investment cost or nearly minimum cost network. Specifically, optimization principles are used to explore analytically the necessary conditions for obtaining minimum investment cost networks. To simplify the analytical developments, the investment costs of exchangers, heaters, and/or coolers are initially approximated as linear functions of their heat transfer areas, and the following auxiliary minimum area or nearly minimum cost network synthesis problem is first examined.

Table 2. Design Data

	4SP1	All other problems
Steam:		
Pressure (lb/in.^{2}abs)	962.5	450.0
Latent heat (Btu/lb)	656.6	767.5
Temperature (°F)	540	456
Cooling water:		
Temperature	100°F	
Heat capacity	1.0 Btu/lb °F	
Maximum water output temperature	180°F	
Minimum allowable approach temperatures:		
Heat exchanger	20°F	
Steam heater	25°F	
Water cooler	20°F	
Overall heat transfer coefficients:		
Heat exchanger	150 Btu/hr ft^2 °F	
Steam heater	200 Btu/hr ft^2 °F	
Water cooler	150 Btu/hr ft^2 °F	
Equipment down time	380 hr/yr for 4SP1 and 6SP1 260 hr/yr for all other problems	
Network cost parameters	$a = 350 \quad b = 0.6$	
Annual rate of return	$\delta = 0.1$	
Cooling water cost	5×10^{-5} \$/lb	
Steam cost	1×10^{-3} \$/lb	

Minimum Area or Nearly Minimum Cost Network Snythesis Problem. For a given total heat duty and properties associated with hot or cold, process and utility streams, find the structure of the network of exchangers, heaters, and/or coolers, and the allocations of heat duty in the network so as to minimize the total investment cost or total heat transfer area. Assume that the investment costs of exchangers, heaters, and coolers can be approximated as linear functions of their heat transfer areas.

The approach is an extension of the previous work of Nishida et al. (1971), in which a similar linear relationship between the area of an exchanger and its investment cost was used to synthesize a network of a minimum investment cost and to determine an optimal amount of heat exchange among hot and cold streams. However, the additions of heaters and coolers served only to satisfy the design specifications. The necessary conditions derived from solving this auxiliary synthesis problem suggest a very simple and practical algorithm for the synthesis of the whole network of exchangers, heaters, and/or coolers as an integrated systems, with the optimal locations of heaters and coolers also determined. For convenience, this synthesis algorithm, which minimizes the total heat transfer area of exchangers, heaters, and coolers based on the linearized investment cost functions will be called *the minimum area algorithm*, and the corresponding network will be referred to as the *minimum area network*. It will be shown that when the actual investment and utility operating costs of the minimum area networks for the example problems are calculated using the original cost expression as a nonlinear function of the heat transfer areas, Equation (1), the resulting total costs of networks are very close or even cheaper than those minimum costs of the similar or different networks obtained by previous synthesis methods. Therefore, in a given heat exchanger network synthesis

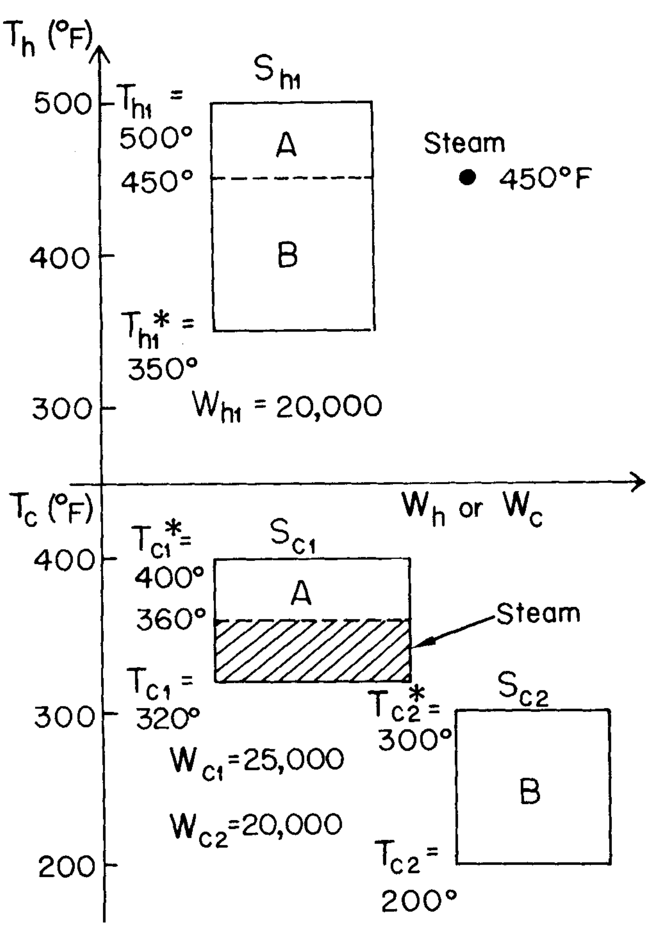


Fig. 1. Step 1 of the minimum area algorithm: representation of the synthesis problem by the heat content diagram.

problem, the minimum area network will be one of the most promising candidates of the minimum cost network.

The next step of the proposed synthesis strategy is to employ a set of rules to modify systematically the initial network created by the minimum area algorithm so that the total cost of investment and utility can be reduced. This is essentially an evolutionary synthesis technique. Since the major emphasis of the present work is to create a network of a minimum investment cost, the evolutionary rules used here will attempt to reduce the investment cost. The main idea behind the evolutionary rules to be described is actually very simple and can be illustrated clearly from the relationship expressing the investment cost as a function of heat transfer areas of exchangers, heaters, and coolers. For example, since the cost factor b in the investment cost expression of exchangers $C_{Hi} = a\,A_{Hi}^{b}$ is normally taken as 0.6, the following two inequalities can be written:

$$a(A_{H1}^{b} + A_{H2}^{b} + \ldots + A_{Hm}^{b}) \geq a(A_{H1} + A_{H2} + \ldots + A_{Hm})^{b} \quad 0 \leq b \leq 1 \quad (2)$$

$$a(A_{H1}^{b} + A_{H2}^{b} + \ldots + A_{Hm}^{b}) \geq a(A_{H1} + A_{H2})^{b} + a(A_{H3} + \ldots + A_{Hi})^{b} + a(A_{H,i+1} + \ldots + A_{Hm})^{b} \quad 0 \leq b \leq 1 \quad (3)$$

These imply that without increasing the total heat transfer area of exchangers, the investment cost of exchangers can be reduced if several exchangers can be combined together as a single one, or a smaller number of exchangers are to be used. Here, Equation (2) corresponds to the case where m exchangers are reduced to a single exchanger; Equation (3) represents the case where m exchangers are reduced to three exchangers. Obviously, the same idea is also applicable in reducing the investment cost of heaters or coolers.

A Minimum Area Algorithm for the Synthesis of A Nearly Minimum Cost Network

The synthesis of the minimum area network can be done by extending a simple, auxiliary graphical representation of the synthesis problem, called the *heat content diagram* (Nishida et al., 1971) to include the use of utilities and by systematically modifying the diagram which represents the original problem to that of the desired minimum investment cost network. In Figure 1, the heat content diagram representing a simple synthesis problem consisting of one hot stream S_{h1}, two cold streams S_{c1} and S_{c2}, and one steam utility stream is shown. In general, the vertical axis of the diagram represents the input and output temperatures of hot and cold process streams, or the input and output temperatures of heating and cooling utility streams. The origin of the temperature scale is set separately such that all hot streams and heating utilities are located above the horizontal axis and all cold streams and cooling utilities below the horizontal axis. The horizontal axis represents the relative magnitude of heat capacity flow rates of various streams. On the diagram, each stream is represented by a block. The area of a given block corresponds to the amount of heat to be removed from or added to the stream in order for it to reach its desired output temperature. For convenience, both process and utility streams should be drawn on the diagram such that heating utilities and hot streams are located in a decreasing order of their input temperatures above the horizontal axis; while cooling utilities and cold streams are located in a decreasing order of their output temperatures below the horizontal axis. In most cases, however, the output temperatures and the heat capacity flow rates of utility streams are both unknown before their exchanges with other process streams are specified. Thus, in representing a given synthesis problem, a heating or cooling utility stream is denoted initially by a point on the diagram, with the ordinate value specifying its known input temperature. For example, a steam stream available at 450°F to be used as a heating utility is shown in Figure 1 as a point above the horizontal axis. Also, on the diagram, a heat exchange between a hot and a cold process stream is indicated by assigning the same number or letter to the corresponding hot and cold blocks as illustrated by letters A and B in Figure 1. Note that since the heat contents of hot and cold process streams to be exchanged must be the same, the hot and cold blocks designated by the same numbers or letters on the diagram must have the same area. Likewise, both above and below the horizontal axis of the diagram, the total number of hot and cold blocks representing the exchanged hot and cold process streams as well as their corresponding total heat contents should also be the same. Finally, division of a block either horizontally or vertically is permitted on the diagram. The former corresponds to the multiple heat exchange, and the latter represents the stream splitting (Nishida et al., 1971). Both will be illustrated shortly.

Based on the above discussion, the following theoretical results can be obtained.

Theorem: If a network of exchangers, heaters, and coolers is the solution of the auxiliary minimum area network synthesis problem, or is a nearly minimum cost network, then the heat content diagram of the network has the following characteristics.

1. The hot process and utility streams in hot blocks and the cold process and utility streams in cold blocks are to be matched consecutively in a decreasing order of their stream temperatures.

2. By labeling the blocks in a decreasing order of their input temperatures, the output temperature of the i^{th} hot block is never lower than the input temperature of the $(i+1)^{th}$ hot block. Similarly, the input temperature of the i^{th} cold block is never lower than the output temperature of the $(i+1)^{th}$ cold block.

From this theorem, the following corollaries, which specify several explicit rules for matching process and/or utility streams in the minimum area or nearly minimum cost network, can be presented:

Corollary 1: Provided that a steam heater is introduced in a network, and that the steam temperature is higher than the highest input temperature of hot streams, the steam heater is placed at the end of the network. Provided that the water temperature is lower than the lowest input temperature of cold streams, the water cooler is placed at the end of the network.

Corollary 2: If a synthesis problem is represented graphically on the heat content diagram, the maximum amount of heat exchange among process streams Q_h can be determined as follows.

1. If $T_{ck} + \Delta T_m > T^*_{hk}$, delete the hot block(s) below the temperature of $T_{ck} + \Delta T_m$, which cannot be exchanged by the cold block(s) owing to the minimum allowable approach temperature. Here, T_{ck} is the lowest input temperature of cold blocks, ΔT_m the minimum allowable approach temperature, and T^*_{hk} the lowest output temperature of hot blocks. If $T_{hk} - \Delta T_m < T^*_{ck}$, delete the cold block(s) above the temperature of $T_{hk} - \Delta T_m$, which cannot be exchanged by the hot block(s). Here, T_{hk} is the highest input temperature of hot blocks and T^*_{ck} the highest output temperature of cold blocks.

2. After the hot and/or cold blocks noted in the above step are deleted, the total heat contents of the rest of the hot and cold blocks, Q'_h and Q'_c, respectively, are to be computed, no matter whether the heat capacity flow rates, W_h and W_c, of hot and cold process streams, respectively, change with the stream temperature or not. If $Q'_h > Q'_c$, Q'_c amount of the heat content of cold streams is to be exchanged with hot streams; and the residual heat content, $\Delta Q = Q'_h - Q'_c$, which cannot be exchanged among hot and cold process streams, can be recovered by the auxiliary cooling utilities. If $Q'_c > Q'_h$, Q'_h amount of the heat content of hot streams is to be exchanged with cold streams, and $\Delta Q = Q'_c - Q'_h$ can be recovered by the auxiliary heating utilities.

Corollary 3: So far as the heat exchange or matching among hot and cold process and utility streams is concerned, the minimum area or nearly minimum cost network has the following characteristics.

1. The hot stream with the highest input temperature is to be matched with the cold stream with the highest output temperature.

2. The hot streams with intermediate input temperatures are to be matched with the cold streams with intermediate output temperatures.

3. The hot stream with the lowest input temperature is to be matched with the cold stream with the lowest output temperature.

The proof of these theoretical results can be done in a similar fashion as in the previous paper (Nishida et al., 1971) and will not be included here.

Corollary 1 and rules 1 to 3 of corollary 3 are very important not only in providing the theoretical guidance on the optimal heat exchange or matching of hot and cold process streams, but also in determining the optimal locations of heating and cooling utilities. Furthermore, matching rules 1, 2, and 3 are to be applied *together* in all synthesis problems, with rule 1 being applied first, rule 2 next, and rule 3 last. The implications of these results can be illustrated with the simple synthesis problem consisting of S_{h1}, S_{c1}, and S_{c2}, and one steam utility stream shown in Figure 1. Here, the input temperature of the steam (450°F) is lower than that of S_{h1} (500°F). Thus, corollary 3 suggests that the highest block of S_{h1} is exchanged first with the highest cold block of S_{c1}, with both being labeled by A. This exchange continues until the output temperature of the highest hot block reaches the steam temperature 450°F. Next, since corollary 3 implies that the steam utility stream is to be matched with cold process streams as if it were a member of hot process streams, the steam is used to heat up the lower cold block of S_{c1} (hatched block). Finally, the lower cold portion of S_{h1} is matched with S_{c2}, with both being labeled B. Based on the latent heat of steam at 450°F and other design data given in Table 2, the computed investment cost of this network, $\sum_{i=1}^{2} 350\ A_{Ei}^{0.6} + 350\ A_{H1}^{0.6}$, is \$7 967/yr. This corresponds to a minimum heat transfer area of 116.2 ft^2. Next, an alternative heat exchange or matching policy, which violates the theoretical guidance of corollary 3, is used in the same problem. This matching policy is the same one used in most published studies on the synthesis of heat exchanger networks which restricts the exchange or matching of a process stream with heating or cooling utilities to its last matching step. Thus, the cold block of S_{c2} is first matched with the lower hot block of S_{h1}. This matching is exactly the same as the previous one shown in Figure 1, labeled by B. However, instead of matching the steam at 450°F with the lower cold block of S_{c1} as suggested by corollary 3, the highest hot block of S_{h1}, labeled by A in Figure 1 above the horizontal axis, is now matched with the lower cold block of S_{c1}. Finally, the steam at 450°F is exchanged with the highest cold block of S_{c1}. On the heat content diagram shown in Figure 1, the last two matches are equivalent to re-labeling the hatched portion of the cold block S_{c1} by A and the upper portion of the cold block, which is denoted by A, by a hatched block. Based on the same design data, the new network has a larger heat transfer area of 122.8 ft^2 and a higher investment cost of \$8 243/yr as compared with those of the previous minimum area network. This comparison has clearly illustrated the importance of the matching rules of corollary 3 in obtaining the nearly minimum cost network. Therefore, the conventional approach, in which the additions of heaters and coolers served only to meet the design specifications and the optimal locations of heaters and coolers are not determined, will exclude many cheaper networks. This point will be further illustrated later in the results of the 4SP2 problem, where the utility operating cost will also be included.

The use of corollary 2 to identify the maximum amount of heat exchange, among hot and cold process streams, and to determine the heat duty of the heating and/or cooling utilities can be illustrated by the simple synthesis problem represented in Figure 2. Here, $T_{ck} = 90°F$, $T^*_{hk} = 100°F$, $\Delta T_m = 20°F$, $T_{hk} = 300°F$, and $T^*_{ck} = 290°F$. As $T_{ck} + \Delta T_m$ $(= 110°F) > T^*_{hk}$ $(= 100°F)$, the portion of the hot block S_{h1} below 110°F is to be deleted. Also, $T_{hk} - \Delta T_m$ $(= 280°F) < T^*_{ck}$ $(= 290°F)$, the portion of the cold block S_{c1} above 280°F is to be deleted. The total heat contents of the rest of the hot and cold blocks are, respectively, $Q'_h = 5.339 \times 10^6$ Btu/hr and $Q'_c = 8.4607 \times 10^6$ Btu/hr. Since $Q'_c > Q'_h$, Q'_h amount of the heat content of hot streams S_{h1} is to be exchanged with cold streams S_{c1} and S_{c2}.

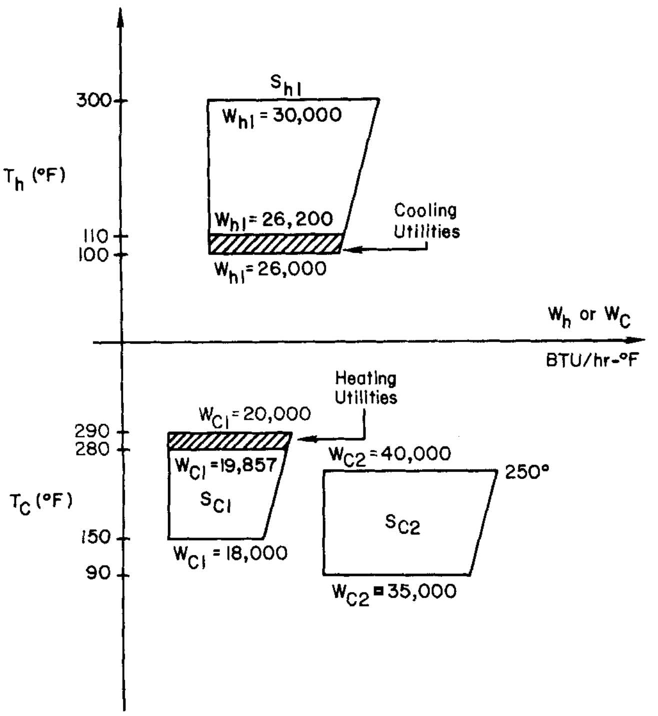


Fig. 2. Step 2 of the minimum area algorithm: identification of the maximum amount of heat exchange among process streams and the heat duty of utility streams according to corollary 2.

Based on the above results and illustrations, an explicit procedure for the synthesis of a minimum area or nearly minimum cost network can now be described as follows.

Step 1. Draw a heat content diagram representing the synthesis problem.

Step 2. Based on the total heat contents of hot and cold process streams, the minimum allowable approach temperature of the exchanger, as well as the input and output temperatures of hot and cold streams, identify on the heat content diagram the maximum amount of heat exchange among process streams. This can be done from the top in hot blocks and from the bottom in cold blocks according to the procedure described in corollary 2. The residual heat content which cannot be exchanged among process streams is to be exchanged with auxiliary utilities. Apply the results of corollary 1 if appropriate.

Step 3. Divide each block horizontally at points where the horizontal edges of the other blocks are located and ignore the distinction of the original block at the same temperature level as shown in Figure 3*a*. This step allows for the use of the multiple heat exchange.

Step 4. Apply the matching rules of corollary 3. Thus, take the highest hot and cold blocks. They are matched to exchange heat after their heat contents are equalized by adopting the high temperature portion of the larger blocks. Repeat this procedure on the unmatched blocks until all blocks are completely matched as illustrated in Figure 3*b*. Note that in Figure 3*b*, the matchings of hot and cold blocks according to rules 1, 2, and 3 of corollary 3 are labeled by 1, 2 and 3, and 4, respectively.

Step 5. If a hot (or cold) block involves several originally distinct blocks, the cold (or hot) block exchanged with this hot (or cold) block is to be divided vertically to match with the distinct blocks as illustrated in Figure 3*b*. This step allows for the use of stream splitting.

Step 6. Draw a flow sheet of the resulting minimum area or nearly minimum cost network by tracing each original stream and its matching on the heat content diagram as illustrated in Figure 3*c*.

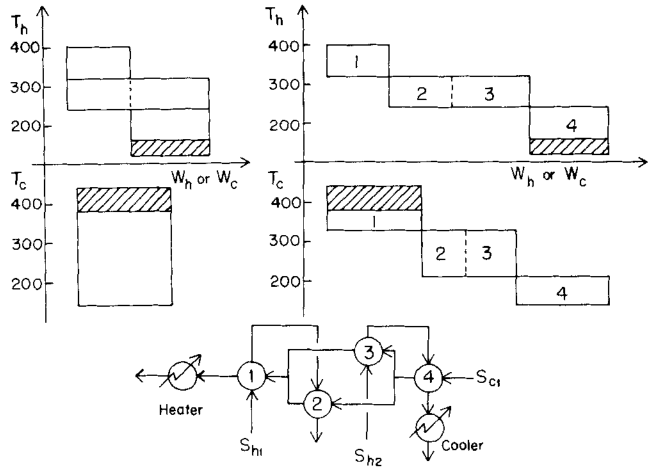


Fig. 3. Steps 3 to 6 of the minimum area algorithm: (a) top left: representation of the use of multiple heat exchange (b) top right: illustration of the use of matching rules 1 to 3 of corollary 3 (c) bottom: flow sheet of the resulting network.

Evolutionary Rules for Improving the Nearly Minimum Cost Network

Based on the evolutionary synthesis strategy described in the preceding section, three effective rules have been developed for improving the nearly minimum cost network obtained by the minimum area algorithm, and they are described as follows.

Rule 1. Compare the areas (heat contents) of various hot and cold blocks in the heat content diagram. Delete the exchanger, the heater, or the cooler with the smallest area or the least amount of heat exchange by increasing the heat duty of the rest of the process and/or utility streams. Repeatedly apply this procedure until the total cost of the resulting network can no longer be reduced.

Rule 2. If a network contains a local sub-network where a hot (or cold) stream matches with the same cold (or hot) stream which it has matched before, delete either of these two repeated matches.

Rule 3. Replace the stream splitting with the stream un-splitting (or the multiple heat exchange). Match the hot and cold streams in the un-splitting network in a decreasing order of their arithmetic averages of input and output temperatures.

It should be noted that these rules are to be applied *in their numerical order;* that is, rule 1 should be tried before rules 2 .to 3, etc. During the course of this work, it has been found that the application of rule 1 always yields improved networks of cheaper total costs based on the cost expression, Equation (1). An example of the block to be deleted according to rule 1 is the cold block labeled by E_1 below the horizontal axis of Figure 7. Furthermore, it has been observed that it is most convenient to apply rule 1 together with steps 3 to 5 in the minimum area algorithm if appropriate, and this will be illustrated in the next section. The successful applications of rules 2 to 3, however, depend on whether or not the computed total cost of the modified network is lower than that of the minimum area network. This fact is, of course, one of the inherent characteristics of any evolutionary synthesis technique.

In Figure 4, a simple network for illustrating the use of rules 2 to 3 is shown. It can be seen from the figure that S_{c1} in network *a* is matched with S_{h1} twice through exchangers 1 and 2. This suggests the use of rule 2 to give a modified network, network *b*. Next, in the latter splitting network, the input temperatures of both S_{c1} and S_{c2} are the same (50°F), but their output temperatures are quite different, with S_{c1} at 400°F and S_{c2} at 150°F. Since the

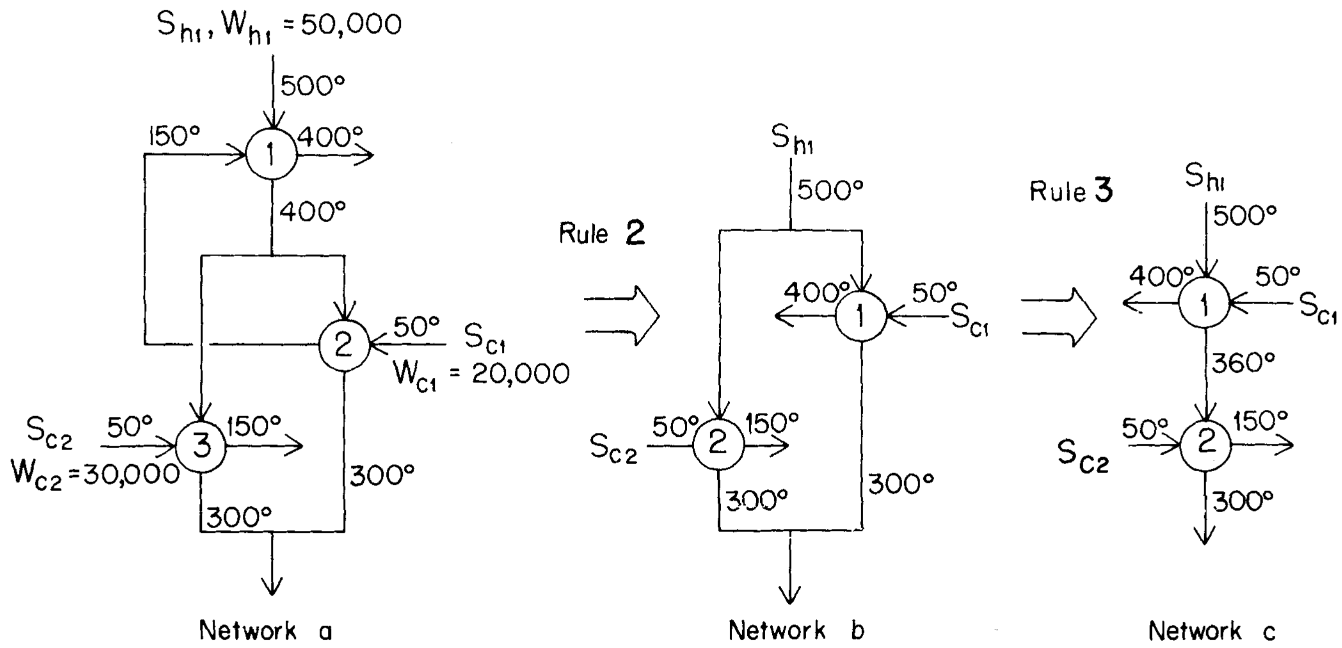


Fig. 4. Illustration of the use of evolutionary strategy, rules 2 to 3.

preceding theoretical results suggest that hot and cold streams are to be matched in a decreasing order of their temperature levels, a convenient way to modify network *b* according to these results is to use the multiple heat exchange or stream un-splitting rather than the stream splitting. This is exactly what rule 3 implies. The remaining question is to determine which one of S_{c1} and S_{c2} is to be matched first with S_{h1} in the resulting un-splitting network. Since the arithmetic averages of the input and output temperatures of S_{c1} and S_{c2} are, respectively, $225°F = (50 + 400)°F/2$ and $100°F = (50 + 150)°F/2$, S_{h1} is thus matched with S_{c1} first and with S_{c2} next, according to rule 3. The corresponding network is represented as network *c* in Figure 4. It should be mentioned that only in the very exceptional cases are the arithmetic averages of the input and output temperatures of hot and/or cold streams the same, so that the proper order of stream matching in the un-splitting network may not be determined by rule 3. One of such exceptional cases has actually been included in network *a* of Figure 4, where the hot residual stream of S_{h1} from exchanger 1 at 400°F is splitted into two parts, with both meeting S_{c1} and S_{c2} of the same average temperature of $100°F = (50 + 150)°F/2$. However, since it has been stated earlier that rule 3 is to be applied after rules 1 to 2 have been employed, such an exceptional case has disappeared after rule 2 is applied, as seen in network *b* of Figure 4. In fact, no such indeterminate cases before rule 3 is applied can be found for all the example problems considered in this work. Finally, by using the design data of Table 2 and the cost expression Equation (1), the computed total annual costs of networks *a, b,* and *c* of Figure 4 are \$17 155, \$14 774, and \$14 754, respectively. The comparison of these costs has clearly illustrated the effectiveness of rules 2 to 3 in the synthesis of nearly minimum cost networks.

ILLUSTRATIVE EXAMPLES

In what follows, the applications of the proposed minimum area algorithm and evolutionary rules to the synthesis of the minimum cost networks for example problems specified in Tables 1 and 2 are illustrated. As will be noted below, there appears to be some discrepancies in the total costs of the optimal networks synthesized by previous investigators, even when the same optimal network is considered. This has been traced to round-off values of stream properties and/or slightly different design data, as well as computational errors. Thus, in this work, all values of total costs of various networks for these examples obtained by previous investigators are re-calculated from the same stream properties and design data given in Tables 1 and 2 so that a valid comparison of different synthesis methods can be made.

Problem 4SP1 (Lee et al., 1970)

The heat content diagram representing the 4SP1 problem is shown as Figure 5. For this problem, Q_h $(= 6.000 \times 10^6$ Btu/hr$)$ $> Q_c$ $(= 5.699 \times 10^6$ Btu/hr$)$. Thus, step 2 of the algorithm suggests that the whole heat content of cold blocks can be exchanged to hot blocks until the highest output temperature of cold blocks T^*_{hk} reaches a temperature $T_{hk} - \Delta T_m = 480 - 20 = 460°F$. Here, T_{hk} is the highest input temperature of hot blocks (480°F of S_{h2}), and ΔT_m is the minimum allowable approach temperature for the exchanger (20°F). In Figure 5, the upper portion of S_{c2} above 460 is thus matched with a heater as

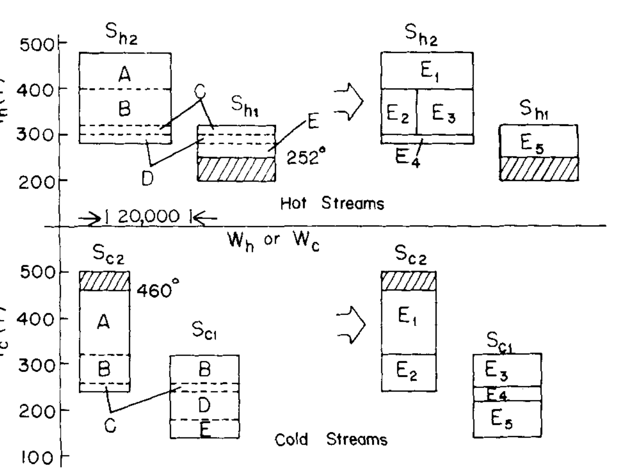


Fig. 5. Representation of the 4SP1 problem.

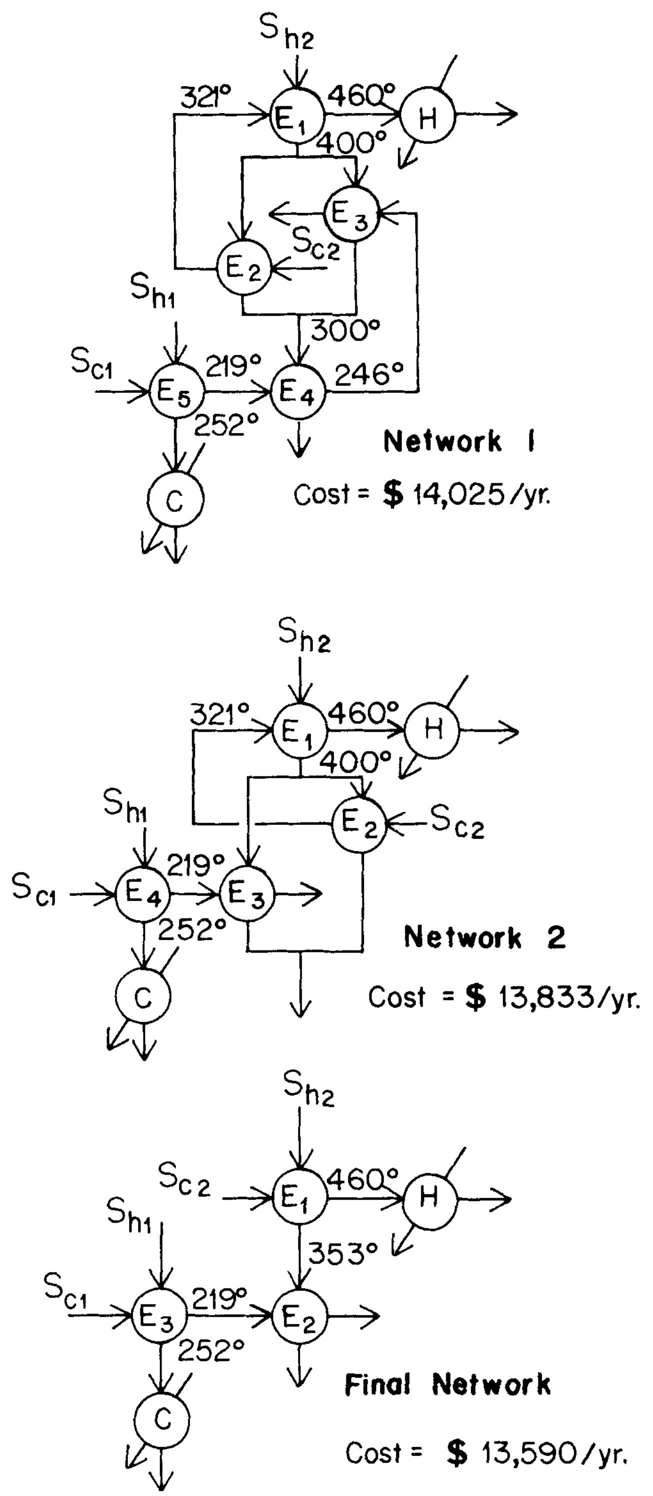


Fig. 6. Nearly minimum cost networks for the 4SP1 problem.

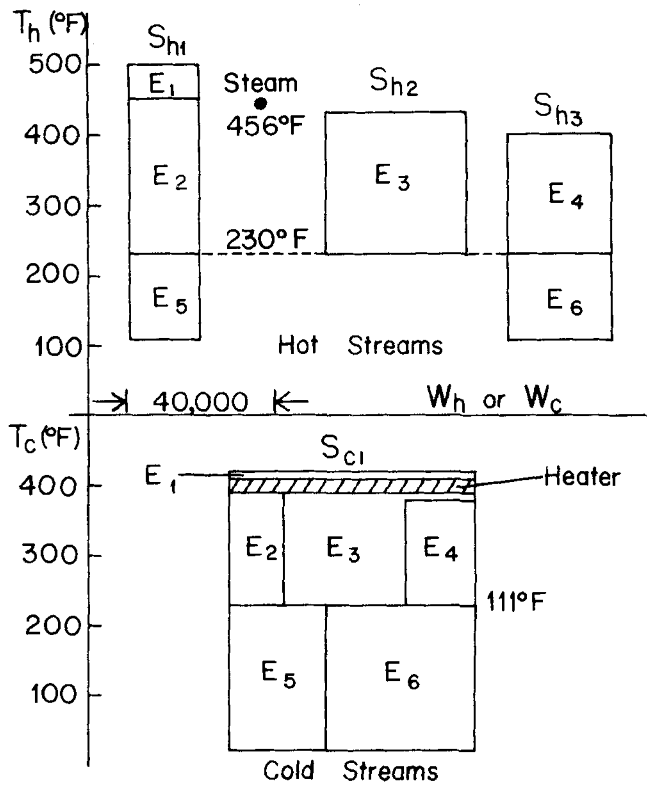


Fig. 7. Representation of the 4SP2 problem.

shown by the hatched block. Next, all hot and cold blocks are matched consecutively in a decreasing order of their stream temperatures according to steps 3 and 4, and the resulting matchings are labeled by letters A, B, C, D, and E in Figure 5. Furthermore, since the hot block B of S_{h2} in Figure 5 is matched with two originally distinct cold blocks labeled by B of S_{c1} and S_{c2}, step 5 suggests that the hot block B of S_{h2} has to be divided vertically to allow for the use of stream splitting. However, as stated earlier, in carrying out such vertical divisions according to step 5, it is convenient to apply rule 1 to delete those matches with small amounts of heat exchange so that the complexity of including exchangers with very small heat transfer areas in the resulting network can be eliminated. Thus, in Figure 5, the small hot block C of S_{h2} is deleted, and the hot block B of S_{h2} is enlarged to cover the heat duty of the deleted block C. The vertical division of the enlarged hot block (B + C) of S_{h2} is now shown in Figure 5 as two distinct blocks labeled by E_2 and E_3, and the remaining hot block D of S_{h2} is also re-designated as E_4. Likewise, hot blocks C, D, and E of S_{h1} above 252°F can be combined to form a single hot block by deleting small hot blocks C and D while enlarging the hot block E. The resulting hot block is shown as E_5 of S_{h1} in Figure 5. Once this simplification is carried out with all hot blocks of S_{h1} and S_{h2} to give hot blocks E_1 to E_5 shown in Figure 5, the corresponding cold blocks to be matched with hot blocks can be identified easily on the heat content diagram according to step 3. The resulting subdivisions of cold blocks of S_{c1} and S_{c2} are also shown in Figure 5, with the corresponding network depicted as network 1 in Figure 6. The total cost of this network is $14 025/yr.

Hereafter, network 1 is modified successively by employing the proposed evolutionary rules. In Figure 5, the heat content of the hot block E_4 is the smallest among all hot blocks, and the application of rule 1 to delete exchanger E_4 and to increase the heat duties of exchangers E_2 and E_3 gives a modified network, network 2, as shown in Figure 6. The total cost of network 2 is $13 833/yr. Network 2 can be further improved by using rules 2 to 3. Since the portion of this network consisting of exchangers E_1, E_2, and E_3 is similar to the previous example illustrated in Figure 4, the final network in Figure 6 can be easily obtained, and its total cost is $13 590/yr. This network is so simple that no further modification will be made. The same network was also found by Rathore and Powers (1975). However, a different network was reported by Lee et al. (1970), McGalliard and Westerberg (1972), and Pho and Lapidus (1973). The annual costs of the network reported by these investigators are $13 481, $13 615, and $13 685, respectively. These costs are slightly different from the value of

\$13 688/yr re-calculated in this work based on Tables 1 and 2. This discrepancy in costs is probably due to round-off values of stream properties and/or slightly different design data, since the values of the latter were not completely specified in the preceding studies.

Problem 4SP2 (Ponton and Donaldson, 1974)

Following the minimum area algorithm, the nearly minimum cost network obtained is represented by the heat content diagram of Figure 7 and shown as network 1 in Figure 8. In Figure 7, it is seen that the hottest portion of S_{h1} with the highest input temperature of 500°F is first matched to the upper portion of S_{c1} until the output temperature of the exchanged portion of S_{h1} reaches 456°F, the temperature of the available steam. Since the steam temperature *is the* second highest input temperature of all hot process and utility streams, the residual heat content of S_{c1} resulted from the last match, 0.115×10^7 Btu/hr, is provided by a steam heater as shown by the hatched block in Figure 7. The total cost of network 1 is \$20 474/yr. Next, since the amount of heat duty by exchanger E_1 is the smallest among all exchangers in network 1 shown in Figures 7, evolutionary rule 1 suggests that an improved network can be obtained by deleting exchanger E_1. This modification gives the final network in Figure 8, with a total cost of \$20 353/yr. Further results obtained for the 4SP2 problem along with their comparison with previous studies have suggested several important implications of different synthesis methods. They are described as follows.

Cyclic and Acyclic Networks. For this problem, Ponton and Donaldson (1974) have applied the matching rule proposed by Nishida et al. (1971) (see further discussion below) and used a branch and bound method. Their solution is shown in Figure 8, and its total cost is \$23 716/yr. These authors have also solved the same problem with a conventional branch and bound method as originally presented by Lee et al. (1970) and found an acyclic network with a total cost of \$72 400/yr, which is more expensive by at least three times than those (\$20 353 to \$23 716/yr) of the cyclic networks shown in Figure 8. This is because in the acyclic network synthesized, significant portions of the heat contents of hot and cold process streams are left to be matched with heating and/or cooling utilities after exchanging among process streams. However, in the cyclic networks generated for the 4SP2 problem, the whole heat content of hot streams can be transferred to that of cold stream, thus resulting with less use of the more expensive utilities. Note that neither the branch and bound method nor the tree searching approach as originally proposed (Lee et al., 1970; Pho and Lapidus, 1973) can generate cyclic networks. The present algorithmic-evolutionary method, the modified branch and bound method of Ponton and Donaldson (1974) and the forward branching tree search method of Rathore and Powers (1975) as well as the heuristic-evolutionary approach of Shah and Westerberg (1975), are the only existing methods that are applicable to the synthesis of cyclic networks. Finally, it may be mentioned that the costs of \$23 716 and \$72 400/yr corresponding to the results of Ponton and Donaldson noted above are the values re-calculated based on the stream properties and design data given in Tables 1 and 2. While the cost of \$23 716/yr is quite close to that of \$23 724/yr reported by Ponton and Donaldson, there is a significant difference between the cost of \$72 400/yr and that of \$63 694/yr calculated by Ponton and Donaldson for the acyclic network obtained by the conventional branch and bound method of Lee et al. This difference is traced to a computational error in the work of Ponton and Donaldson. Thus, the output temperature of the final residual of the cold stream S_{c1} in Figure 2*a*

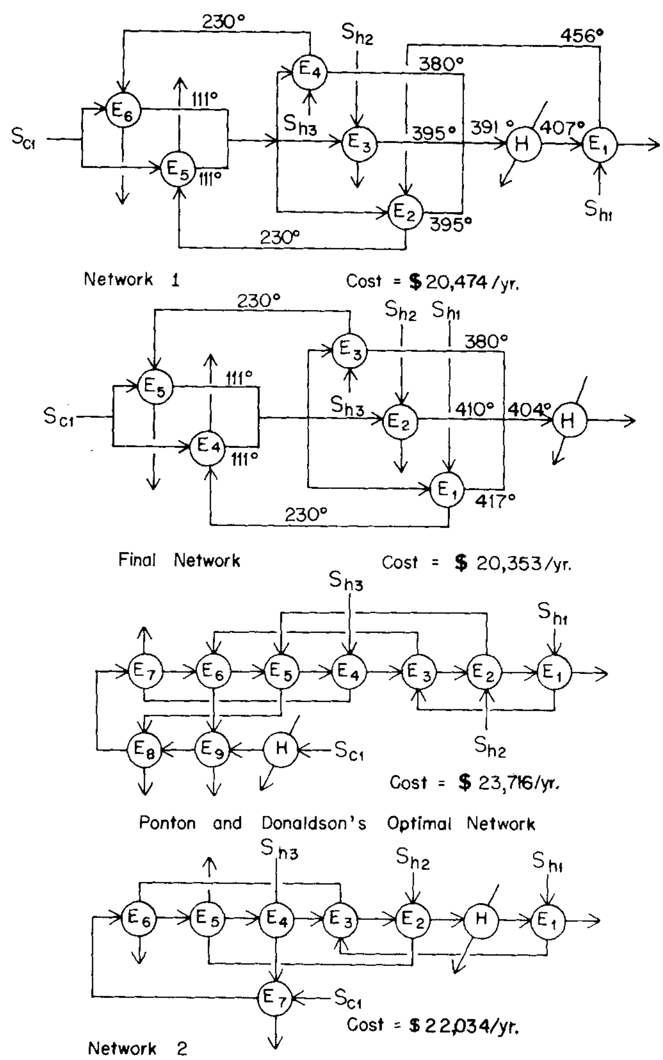


Fig. 8. Nearly minimum cost networks for the 4SP2 problem.

of the paper of Ponton and Donaldson (1974) should be 362.5°F, instead of 370°F.

Optimal Locations of Utilities. The above solutions to the 4SP2 problem also provide an excellent illustration of the significance of corollary 3 in determining the optimal locations of heating and/or cooling utilities in a nearly minimum cost network. For instance, the input temperatures of hot process and utility streams arranged in a decreasing order of magnitude are $T_{h1} = 500°F$, $T_s = 456°F$, $T_{h2} = 430°F$, and $T_{h3} = 400°F$. Thus, the steam heater should be placed between the two exchangers E_1 and E_2 in which S_{h1} and S_{h2} are matched with S_{c1}, respectively. However, as seen in Figure 8, the location of the steam heater in the optimal network obtained by Ponton and Donaldson is not consistent with this theoretical guidance and thus will exclude other networks of less total costs. To illustrate this point clearly, an alternative network synthesized essentially by the procedure of Ponton and Donaldson with the exception of placing the steam heater according to the above theoretical guidance is shown as network 2 in Figure 8. The total cost of this modified network, \$22 034/yr, is now indeed cheaper than that of the optimal network of Ponton and Donaldson, \$23 716/yr. Note that when the method of Ponton and Donaldson is applied, heaters are always placed at one end of the network, except for the case where the temperature of the hottest hot stream is much lower than the highest output temperature of cold streams; while coolers are always placed at the other end of the network. Like-

TABLE 3. COMPARISON OF DIFFERENT NETWORKS FOR THE 4SP2 PROBLEM

Network	Total heat transfer area, ft^2	Total investment cost ($/yr)	Total utility operating cost ($/yr)	Total cost ($/yr)
Network 1, Figure 8	2 519	7 738	12 736	20 474
Final network, Figure 8	2 696	7 617	12 736	20 353
Optimal network of Ponton and Donaldson (1974), Figure 8	3 760	10 980	12 736	23 716
Network 2, Figure 8	3 067	9 298	12 736	22 034
Acyclic network obtained by the branch and bound method of Lee et al. (1970), see Ponton, and Donaldson (1974)	3 263	8 633	63 767	72,400

wise, in most of other previous studies on the synthesis of heat exchanger networks, it has always been assumed that if a process stream does not reach its desired output temperature, it reaches its output temperature by an exchange with a utility stream in an auxiliary heater or cooler. The exchange or matching of a process stream with heating or cooling utilities is thus restricted to its last matching step. This approach, however, will exclude many networks of high energy recovery and less total cost as illustrated in the generation of network 2 in Figure 8.

Matching Rules, Minimum Area Network, and Minimum Cost Network. For the 4SP2 problem, the minimum area network generated by applying all of the three matching rules 1 to 3 of corollary 3 shown as network 1 of Figure 8 has a total cost of $20 474/yr. This is much cheaper than that ($23 716/yr) of the optimal network obtained by Ponton and Donaldson depicted also in Figure 8 by applying only matching rule 1 followed by a branch and bound method. This comparison illustrates an important observation that if all of the three matching rules of corollary 3 are applied in the generation of initial networks in many synthesis problems, the resulting initial networks will correspond to some nearly minimum cost networks. The total costs of these initial networks are often so low that there is no need to use any further techniques such as branch and bound method or the evolutionary approach to improve these networks. Further implications of these matching rules are discussed in the solution of other example problems. The heat transfer areas, investment costs, utility operating costs, and total costs of the different networks synthesized are summarized in Table 3. In this table, the acyclic network obtained by the conventional branch and bound method of Lee et al. (1970) has a very expensive utility operating cost, $63 767/yr, while all other networks have the same and much lower utility operating cost of $12 736/yr.

Based on the above results, it is appropriate to comment briefly here on the work of Ponton and Donaldson (1974) as well as on its connection with the earlier paper of Nishida et al. (1971) and the present study. In the paper of Ponton and Donaldson, the following statement related to the matching rule 1 of corollary 3 and the above results was given:

"This matching of hottest hot stream with highest cold target *is* the heuristics on which the present method is based."

"This intuitive procedure has some theoretical justification. Nishida et al. (1971) have shown that with certain restrictions, the network in which hottest hot and cold streams are paired at the inlet to each exchanger will have minimum total exchanger area for a given total heat duty. *This is not, however, necessarily the same as matching hottest hot stream with highest cold target, nor does the minimum area network in general correspond to the minimum cost network when heaters or coolers are required.*"

However, a careful review of the paper of Nishida et al. (1971) and the results of the present study will suggest that the above italicized statement made by Ponton and Donaldson is not true. It should be pointed out that the matching of hottest hot stream with highest cold target was clearly first proposed and applied in the earlier paper of Nishida et al. (1971). Specifically, in the network synthesis procedure described on page 1848 of the latter paper, the following was stated: "*Take the highest hot and cold blocks. They are matched to exchange heat after their heat contents are equalized by adopting the high temperature portion of the larger blocks.* Repeat this procedure on the unmatched blocks until all the blocks are completely matched." The extension of this procedure to include the determination of the optimal locations of heaters and coolers leads to step 4 of the present minimum area algorithm along with the theoretical guidance of the preceding theorem and corollaries 1 to 3. Next, the results of this study have clearly indicated the fact that when the synthesis strategy is to achieve a maximum amount of heat exchange among hot and cold process streams as used in this work as well as in many previous studies by Lee et al. (1970), Pho and Lapidus (1973), and Ponton and Donaldson (1974), the utility operating cost is practically fixed and almost minimum. Consequently, even when heaters or coolers are required, the minimum area network, which is also usually the minimum investment cost network, will generally correspond to the minimum cost network. Indeed, the results for the 4SP2 problem summarized in Table 3 have clearly shown that the utility operating costs for the first four networks in Figure 8 are all the same ($12 736/yr), and the minimum area or minimum investment cost network does correspond to the minimum cost network.

Splitting and Un-splitting Networks. For certain synthesis problems in which the capacity flow rate of any cold (or hot) stream is markedly greater than those of hot (or cold) streams, the use of stream splitting will generally lead to networks of high energy recovery and low investment cost, and evolutionary rule 3 will preferably *not* be applied. For instance, in the 4SP2 problem, the capacity flow rate of S_{c1} ($70 000 Btu/hr-°F) is greater than those of S_{h1}, S_{h2}, and S_{h3} ($20 000 to 50 000 Btu/hr-°F). The comparison of the first four networks of Table 3 shown in Figure 8 shows that the total cost of the final network which is the cheapest splitting network ($20 353/yr) is indeed lower than that ($22 034/yr) of the cheapest unsplitting network, network 2. This difference in total costs is obviously a result of the decrease in the total investment cost due to the decrease in the total heat transfer area when stream splitting is allowed. Actually, the 4SP2 problem is quite similar to many large scale industrial heat exchanger

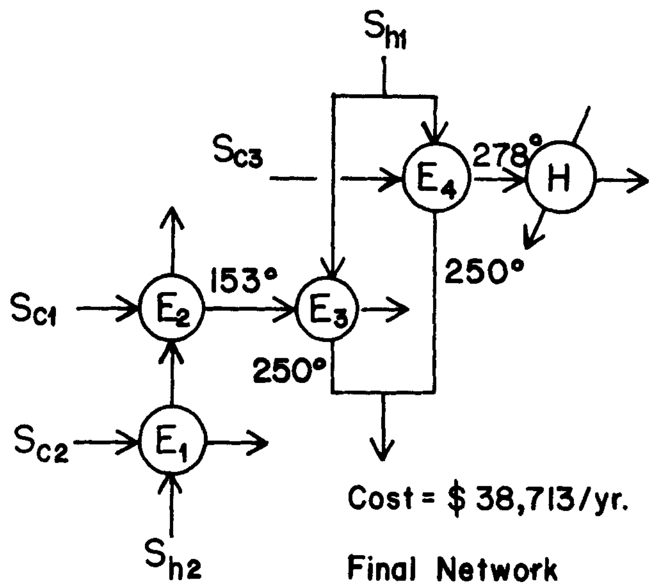


Fig. 9. Final network for the 5SP1 problem.

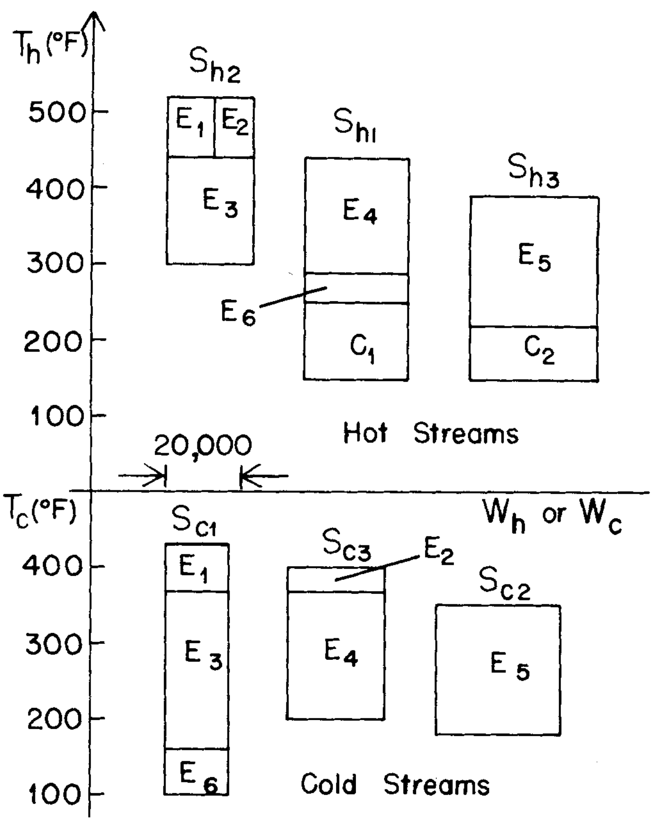


Fig. 10. Representation of the 6SP1 problem.

network synthesis problems. One typical example is the crude preheat exchanger network around the topping tower in the common petroleum refinery plant (Hwang and Elshart, 1976), in which one cold crude oil stream is used to cool down six to ten hot distillate streams. The capacity flow rate of the crude oil stream is about the sum of the corresponding rates of all hot distillates. However, except for the synthesis techniques described in Kobayashi et al. (1971), Nishida et al. (1971), Shah and Westerberg (1975), and Hohmann and Lockhart (1976) as well as this study, all previous methods have been restricted only to the case with no stream splitting.

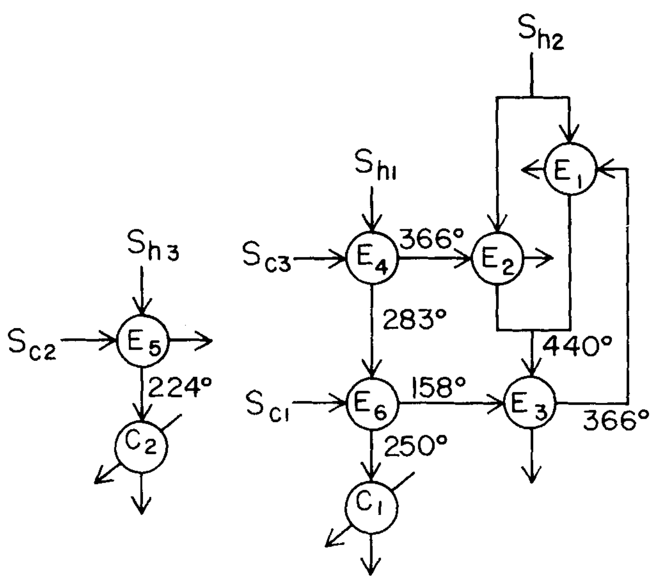
Problem 5SP1 (Masso and Rudd, 1969)

Following the proposed synthesis procedure, the cheapest splitting network obtained is shown as the final network in Figure 9, with a total cost of $38 713/yr. Another nearly minimum cost network synthesized with no stream splitting has the same structure as Figure 7 of the paper of Masso and Rudd (1969) and the one found by Pho and Lapidus (1973). Briefly, this network is similar to that of Figure 9, except that S_{h1} is first matched with S_{c3} and with the residual portion of S_{c1} leaving exchanger E_3, and finally a heater is placed at the residual portion of S_3 leaving exchanger E_3. The total cost of this un-splitting network, $38 762/yr, is slightly higher than that of $38 713/yr of the preceding splitting network. Note that the total cost of $38 762/yr for the un-splitting network is also slightly different from the values of $38 927/yr reported by Masso and Rudd (1969) and of $38 268/yr calculated by Pho and Lapidus (1973) for the same network structure. This discrepancy is probably due to the use of different design data such as the latent heat of steam at 456°F and 450 lb/in.^{2} abs. In this work, it has been found that a small difference of 0.5 Btu/lb in the latent heat of steam used will lead to a corresponding difference in the total cost of the network of the order of $20 to $80/yr.

For the 5SP1 problem, if the method of Ponton and Donaldson (1974) is applied, the resulting network will generally include more heaters and/or coolers than is necessary, and the corresponding total cost is quite sensitive to the matching rule used in generating the initial network needed for their branch and bound method. For example, by using the initial network generated by the matching rule 1 of corollary 3, and following the procedure of Ponton and Donaldson, the final network obtained includes three heaters and one cooler with a total cost of $45 199/yr. Alternatively, if the matching rule 3 of corollary 3 is used to generate the initial network, the resulting network by the method of Ponton and Donaldson includes two heaters with a total cost of $39 057/yr. Since both of these networks are more expensive than the minimum cost network shown in Figure 9 ($38 713/yr), their details will not be given here. The above solutions to this problem, however, have illustrated another important implication of the results of this study. Thus, in certain synthesis problems like the 5SP1 problem, if any of the three matching rules 1 to 3 of corollary 3 is applied alone to generate the initial networks followed by some other techniques to improve these initial networks, the total costs of the final networks obtained are often quite sensitive to the initial matching rule used. Consequently, if proper minimum cost networks are to be obtained, it is important to use *all* of the three matching rules 1 to 3 of corollary 3 together in the generation of any initial networks, no matter whether the subsequent modification technique is the branch and bound method or the evolutionary approach.

Problem 6SP1 (Lee et al., 1970)

By applying the minimum area algorithm and evolutionary rule 1, the nearly minimum cost network obtained is represented by the heat content diagram of Figure 10 and shown as network 1 of Figure 11, with a total cost of $35 451/yr. Next, the application of evolutionary rules 2 to 3 gives an improved network of a total cost of $35 010/yr depicted as the final network in Figure 11. This network is so simple that no further modification will be made. The total cost of the final network ($35 010/yr) in Figure 11 is cheaper than the different networks synthesized by previous investigators such as Lee et al. (1970), McGalliard and Westerberg (1972), and Pho and Lapidus (1973). The corresponding costs of the network obtained by these authors are $37 331/yr, $35 780/yr, and $35 659/yr, respectively.



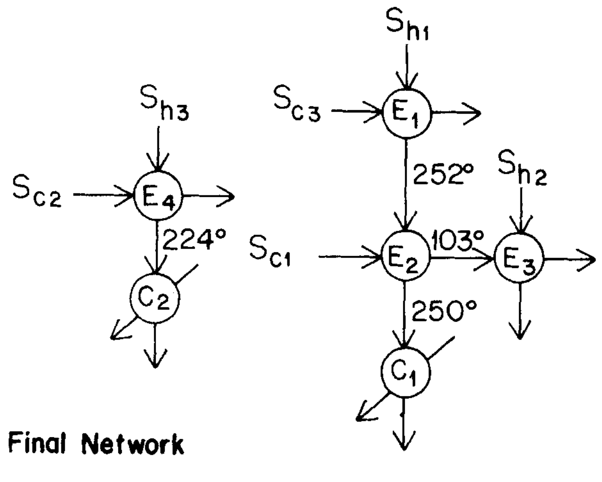


Fig. 11. Nearly minimum cost networks for the 6SP1 problem.

Note that the total cost of the network obtained by Lee et al. (1970) as given here, \$37 331/yr, is the correct value calculated based on Tables 1 and 2. This value was previously given incorrectly as \$35 108/yr by Lee et al. and \$35 714/yr by Pho and Lapidus. Although a computational error in the work of Lee et al., namely, the first residual of S_{c2} should be 243°F instead of 230°F, was found earlier by Pho and Lapidus, there still exists another computational error. Subject to the constraint of the minimum allowable approach temperature, the amount of cooling water required for the cooler placed at the final residual of S_{h1} should be computed based on an output temperature of water at 157°F, instead of 180°F as in these previous studies.

Problem 10SP1 (Pho and Lapidus, 1973)

To illustrate the effectiveness of the present synthesis method in solving large scale problems, the ten-stream problem proposed by Pho and Lapidus (1973) is considered here. As discussed by Pho and Lapidus, the 10SP1 problem represents the maximum possible size for any problems with ten hot and cold streams. The applications of other synthesis techniques are generally prohibited owing to the combinatorial difficulties associated with these techniques.

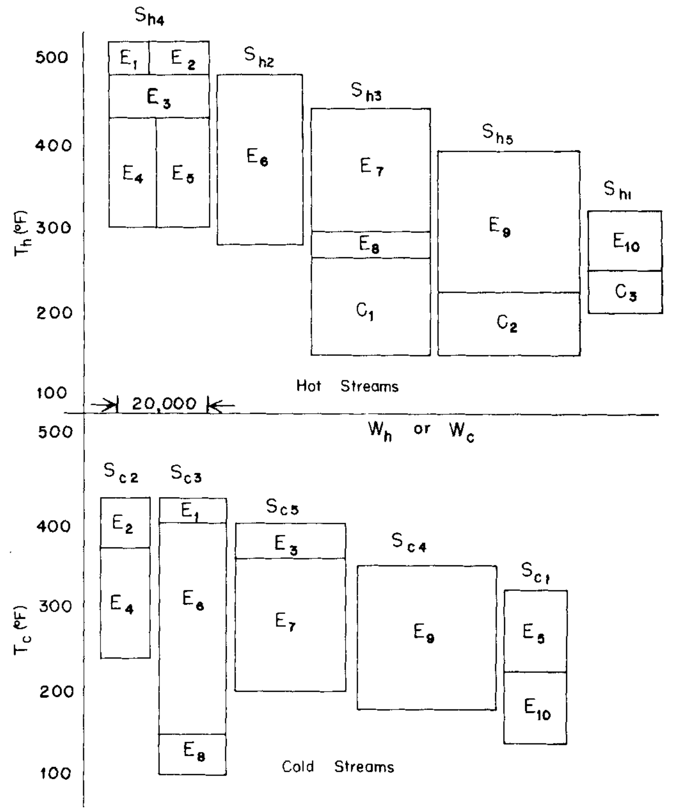


Fig. 12. Representation of the 10SP1 problem.

By applying the minimum area algorithm, the initial, nearly minimum cost network obtained is represented by the heat content diagram of Figure 12 and shown as network 1 of Figure 13. The total cost of network 1 is \$44 778/yr. In Figure 12, the areas of both hot blocks for exchanger E_1 and cooler C_3 are smaller as compared with other blocks, thus suggesting the application of evolutionary rule 1. Consider first the deletion of cooler C_3. This can be accomplished by increasing the heat duty of either cooler C_1 or C_2. If cooler C_1 is to be enlarged, the coldest portion of S_{c3}, which currently meets the residual portion of S_{h3} through exchanger E_8, is to be matched with the coldest portion of S_{h1} that is exchanged with cooler C_3. This modification essentially follows the theoretical guidance of the matching rule 3 of corollary 3, and the resulting network after cooler C_3 is deleted is shown as network 2 of Figure 13. Next, exchanger E_1 is deleted by increasing the heat duty of the rest of the exchangers that are connected to exchanger E_1 in network 2. This leads to an improved network, network 3, of less total cost (\$44 247/yr) shown in Figure 14. Further modifications of network 3 can be done by requiring S_{h4} and S_{c2} to be matched only once and by un-splitting the local splitting sub-network consisting of exchangers E_4 and E_5 according to evolutionary rules 2 to 3. Since S_{h4} is matched with S_{c1}, S_{c2} and S_{c5} through three exchangers in network 3 of Figure 14, the proper order of their matches in the un-splitting network can be determined by comparing the arithmetic averages of input and output temperatures of S_{c1}, S_{c2} and S_{c5} in network 3. This comparison suggests that S_{h4} is first matched with S_{c5}, next with S_{c2}, and finally with S_{c1}. The modification gives the final network of Figure 14, with a total cost of \$43 984/yr. This network is cheaper than a different final network of a total cost of \$44 160/yr obtained by Pho and Lapidus, and no further modifications will be made.

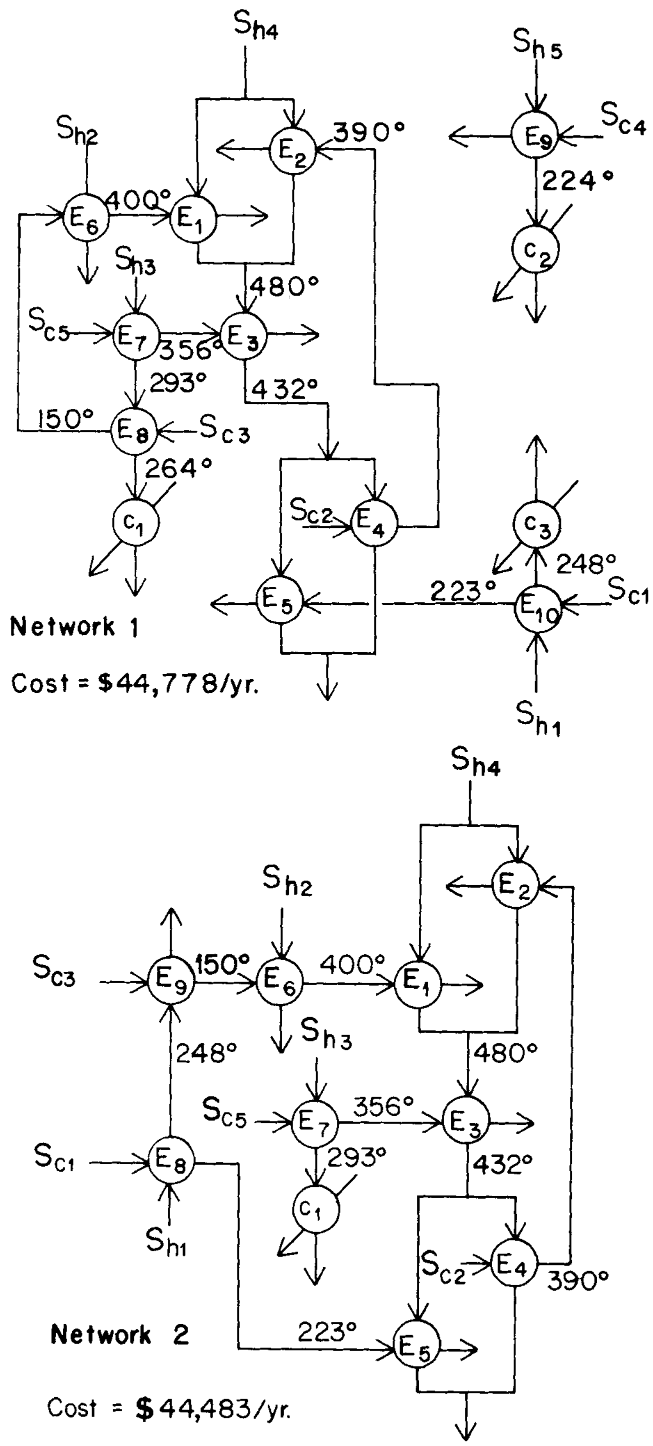


Fig. 13. Networks 1 and 2 for the 10SP1 problem.*

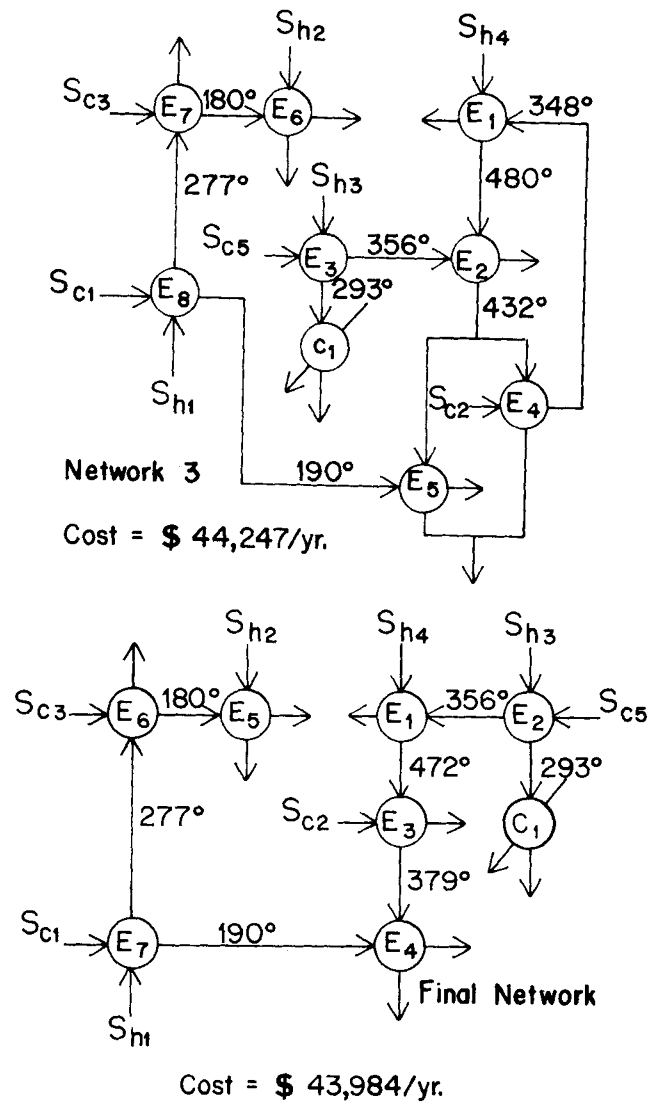


Fig. 14. Network 3 and final network for the 10SP1 problem.*

DISCUSSION

Further Comparison with Previous Studies

As noted in the preceding developments and examples, there are several important differences of the proposed method from those by other investigators. The new method is easy to apply by hand calculations, requiring no special mathematical background and computational skill from the user. While the computer automation of the new method is definitely possible, it is not necessarily needed in the synthesis of large-scale heat exchanger networks. This has been clearly illustrated in the solution of the 10SP1 problem. The proposed method provides an explicit theoretical guidance on the optimal exchange among hot and cold streams and the optimal locations of heating and cooling utilities in the network. It also has a provision for the use of stream splitting and for generating a cyclic network. All of these aspects can be further illustrated in Table 4, in which a detailed comparison of the present work with previous studies is given

It is worthwhile to comment more clearly on the connections and differences between the present study and those of Nishida et al. (1971), Shah and Westerberg (1975), and Hohmann and Lockhart (1976). The minimum area algorithm presented in this paper has at least two major changes from the previous one by Nishida et al. (1971). First, the new algorithm does not require the iterative computer calculations in optimally coordinating the heat duty of the interior subsystem (heat exchangers only) and the external subsystem (heaters and/or coolers) as in Nishida et al. Instead, the maximum amount of heat exchange among hot and cold process streams is always determined by corollaries 1 and 2 to give the required heat duty. Next, the present algorithm allows utility streams to be considered within the network rather than at the periphery only; and the optimal locations of heaters and coolers in the network are also determined. This aspect represents another important difference not only from the previous work of Nishida et al. but also from all existing publications as noted in Table 4.

In a recent paper by Shah and Westerberg (1975), a three-step evolutionary approach for the systematic gen-

* For simplicity, the exchange between S_{h5} and S_{c4} in network 2 of Fig. 13 as well as in network 3 and final network of Fig. 14 is not shown explicitly. This exchange is identical to that included in network 1 of Fig. 13.

TABLE 4. COMPARISON WITH PREVIOUS STUDIES

Reference	Type of streams in examples	Optimal locations of utilities determined	Allowing stream splitting	Explicit matching rules given or applied	Generating cyclic network	Network investment cost function	Largest problem solved	Computer program necessary	Synthesis technique
1. Hwa (1965)	Liquid streams only	No	No	No	No	$A^{1.0}$	No example	Yes	Separable programming
2. Kesler and Parker (1969)	Liquid streams only	No	No	No	No	$A^{1.0}$	6 streams	Yes	Linear programming
3. Masso and Rudd (1969)	Liquid streams only	No	No	No	No	$A^{0.6}$	7 streams	Yes	Heuristic structuring
4. Let et al. (1970)	Liquid streams only	No	No	No	No	$A^{0.6}$	7 streams	Yes	Branch and bound
5. Kobayashi et al. (1971)	Liquid streams only	No	Yes	Applied	Yes	$A^{1.0}$	7 streams	Yes	Iterative linear programming
6. Nishida et al (1971)	Liquid streams only	No	Yes	Given	Yes	$A^{0.6}$	9 streams	Yes	Algorithmic
7. Menzies and Johnson (1972)	All phases phase change	No	No	No	No	$A^{0.6}$	9 streams	Yes	Branch and bound plus heuristic
8. McGalliard and Westerberg (1972)	Liquid streams only	No	No	No	No	$A^{0.6}$	6 streams	Yes	Evolutionary search with bounding
9. Pho and Lapidus (1973)	Liquid streams only	No	No	No	No	$A^{0.6}$	10 streams	Yes	Tree search
10. Sirrola (1974)	Liquid or gas streams/phase change	No	No	Given	No	$A^{0.6}$	13 streams	Yes	Tree search
11. Ponton and Donaldson (1974)	Liquid streams only	No	No	Applied	Yes	$A^{0.6}$	10 streams	Yes	Heuristic plus branch and bound
12. Rathore and Powers (1975)	Liquid streams only	No	No	Given	Yes	$A^{0.6}$	4 streams	Yes	Tree search
13. Shah and Westerberg (1975)	Liquid streams only	No	Yes	Applied	Yes	$A^{0.6}$	6 streams	Yes	Heuristic and evolutionary
14. Kelahan and Gaddy (1976)	Liquid streams only	No	No	Applied	No	$A^{0.6}$	6 streams	Yes	Mixed integer programming
15. Hohmann and Lockhart (1976)	Liquid streams only	No	Yes	Given	No	$A^{0.6}$	8 streams	Yes	Heuristic and algorithmic
16. This work	Liquid streams only	Yes	Yes	Given	Yes	$A^{0.6}$	10 streams	No	Algorithmic and evolutionary

eration of minimum cost networks of exchangers, heaters, and coolers has been presented. The approach begins with the synthesis of an initial nearly minimum cost network by, for example, matching the hottest hot with the hottest cold process streams as originally proposed by Nishida et al. (1971). Note that in the present work, three matching rules, rules 1 to 3 of corollary 3, are proposed for generating the initial networks. These matching rules are extensions of those presented previously by Nishida et al. (1971) in considering the synthesis of the whole network of exchangers, heaters, and coolers as an integrated system, with the optimal locations of heaters and coolers also determined. Further, it has been shown in this work that if proper minimum cost networks are to be obtained, it is important to use all of the three matching rules in the generation of initial networks. Thus, these aspects represent the major differences between the present approach and that of Shah and Westerberg in the synthesis of initial networks. The next step of the approach of Shah and Westerberg involves the analysis of the neighboring network structures generated by applying a set of evolutionary rules to modify the initial network and the identification of the minimum cost neighboring networks for subsequent optimal design studies. Four evolutionary rules, designated here as rules 1*a* to 4*a*, have been suggested by Shah and Westerberg. These include: rule 1*a*—reverse the stream direction through two adjacent exchangers, rule 2*a*—add or delete an exchanger, rule 3*a*—split or un-split a stream through two adjacent exchangers, and rule 4*a*—interchange a mixer and an exchanger. In the work of Shah and Westerberg, only rules 1*a* to 3*a* were applied to example problems, and the actual application involved two stages. First, rules 1*a* and 2*a* were applied for modifying the initial network to give a set of neighboring networks, and the minimum cost neighboring network was identified. Next, rule 3*a* was used to see if further cheaper neighboring networks could be obtained. In some cases, even after rule 3*a* was applied, an originally minimum cost neighboring network might become a nearly minimum cost one. In those situations, Shah and Westerberg suggested that the above two-stage procedure be repeated and the neighboring networks of the original neighboring networks of the initial network be evaluated. The last step of the approach of Shah and Westerberg is to use the complex method to optimize the minimum cost neighboring network obtained by the previous two steps. Here, the optimization minimizes the number of heaters and coolers as well as their corresponding heat duties. It should be noted that there are also a number of significant differences between the above steps of evolutionary analysis and optimization in the approach of Shah and Westerberg and the evolutionary strategy of the present work. First, for the four evolutionary rules proposed by Shah and Westerberg, rules 1*a* and 4*a* are not used at all in the present study; while rules 2*a* and 3*a* have been greatly modified in the present paper. Note that no specific a priori guidance is included in the evolutionary rules of Shah and Westerberg as to whether an exchanger is to be added or deleted, as well as whether stream splitting or unsplitting is to be used, etc. However, the theoretical results of this study, as illustrated by the algebraic inequalities, Equations (2) and (3), have indicated that in order to obtain a modified network of a cheaper total cost, no exchangers should be added. Instead, rule 1 of the present paper suggests that only the exchanger with the least amount of heat duty will be deleted, and this will ensure a cheaper modified network. Further, rule 1 also suggests that the same approach is to be applied to the heater and cooler with the smallest amounts of heat duties in the network to be modified. Next, rule 3 of the present paper suggests that no splitting of an initial network should be done in the evolutionary step of the synthesis, except for those problems in which the capacity flow rate of a cold (or hot) stream is markedly greater than those of hot (or cold) streams as in the 4SP2 problem. Also, an explicit theoretical guidance has been included in rule 3 on the order of stream matching when an originally splitting network is replaced by an un-splitting network. In addition to the new rule 2 proposed, the above aspects represent the major differences between the evolutionary rules in the work of Shah and Westerberg and the present paper. As a result, in applying their evolutionary strategy, it is often necessary to compare the total costs of a large number of neighboring networks so that a minimum cost neighboring network can be identified. For instance, for a synthesis problem with five to six streams, it was reported by Shah and Westerberg that the total costs of about fifteen to thirty neighboring networks had to be evaluated. It may also be noted that in the present method, the utility operating cost is minimized by simply applying corollary 2 and rule 1. This approach appears to be much easier than the optimization step included in the method of Shah and Westerberg. In addition, no attempt has been made in this study to use the optimization approach to find the best set of network design variables such as the stream splitting ratio and the minimum approach temperature of the exchanger, etc., since this optimization is generally more time-consuming. Instead, the synthesis strategy taken here is the one used in such previous studies by Lee et al. (1970) and Pho and Lapidus (1973), emphasizing only the optimization of the network structure variables by the synthesis technique chosen, like the minimum area algorithm, and improving the resulting network structure later by other simpler approaches, such as the evolutionary rules proposed in this paper. Optimization of the network design variables is, perhaps, best left to final design studies with rigorous design equations based on several nearly minimum cost networks and auxiliary performance indexes other than the total cost such as the control consideration, the safety features, the parameter uncertainty, etc. (Nishida, Liu, and Ichikawa, 1975, 1976; Liu et al., 1976). Finally, it is worth noting that Hohmann and Lockhart (1976) have recently reported their results on the computer automation of a minimum area network approach which is similar to that presented first by Nishida et al. (1971). Their work utilizes the temperature-enthalpy diagram and the concept of temperature contention. The latter exists when two or more hot or cold streams, with equal effective heat transfer coefficients, are available for heat exchange over a common temperature interval. These authors have also described the implications of the minimum area, the minimum approach temperature, the minimum and maximum numbers of exchangers, etc., in the synthesis of heat exchanger networks. Since their method is still under developments it is not possible to compare more specifically their method with the present approach. Their results reported, however, have clearly indicated that the minimum area approach as described in this work can be computerized, as was also shown previously by Nishida et al. (1971).

Some Remarks on the Simplifying Assumptions

The simplifying assumptions included in this work are the same ones that have been used in almost all existing studies for the synthesis of heat exchanger networks. Although most of the assumptions are still somewhat idealized, the latest extensions of the present approach have indicated that the new method can be developed for the solutions of more practically oriented synthesis problems. Existing analyses have already shown that the method can be extended to synthesize the network with multiple-pass shell and tube exchangers. The latter has not been studied seriously in the literature. In particular, the introduction of a correction factor to the log mean temperature difference (LMTD) for the case of multiple-tube passes does not affect the validity of a majority of the theoretical results presented in this paper, such as corollaries 1 to 3 and evolutionary rules 1 to 3. For instance, Equations (2) and (3), the inequalities on which rule 1 is based, are valid no matter whether a correction factor is introduced or not. These inequalities are purely algebraic results because of the cost factor $0 \leq b \leq 1$. The correction factor for the case of multiple-tube passes will only change the value of the individual heat transfer area A_{Hi}, but not the validity of the inequalities. Also, the present method can be easily extended to allow for phase changes of process streams. This follows because of the use of the heat content diagram in the method, and it is possible to include additional blocks in the diagram representing the amounts of latent heat of either hot or cold streams. This aspect is being studied by the authors at the present time. Finally, if the effective heat transfer coefficients U's are not equal, the minimum area algorithm can still be applied to generate a nearly minimum cost network. In this case, the three matching rules proposed should only be considered as a set of effective heuristics, instead of rigorous theoretical results. In fact, a special version of the minimum area algorithm in this paper was applied successfully by Nishida et al. (1971) to the synthesis of minimum cost heat exchanger networks for an industrial problem with different values of effective heat transfer coefficients. Further extensions of the present approach for the synthesis of this type of heat exchanger network problem are in progress.

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NOTATION

a, b = constant in network investment cost function, Equation (1) and Table 2

A_{Ei}, A_{Hi}, A_{Ci} = heat transfer areas for the i^{th} heat exchanger, steam heater, and water cooler, respectively, Equation (1)

$C_{Ei}, C_{Hi}\, C_{Ci}$ = investment costs for the i^{th} heat exchanger, steam heater, and water cooler, respectively

J = total investment and utility operating cost of the heat exchanger network, Equation (1)

Q_h, Q_c (Q'_h, Q'_c) = total heat contents of hot and cold process streams, respectively

Q_R = maximum amount of heat exchange among process streams

ΔQ = residual heat content which cannot be exchanged among hot and cold process streams

S_{hi}, S_{cj} = i^{th} hot and j^{th} cold process streams, respectively

S_{uk1} = amount of utility such as steam or water spent at the 1^{th} auxiliary equipment per year, Equation (1)

T = temperature

T_{ck}, T^*_{ck} = lowest input and the highest output temperatures of cold blocks, respectively

T_{hk}, T^*_{hk} = highest input and the lowest output temperatures of hot blocks, respectively

ΔT_m = minimum allowable approach temperature for the heat exchanger

u_k = operating cost of the utility S_{uk} per year, Equation (1)

W_i = heat capacity flow rate of the i^{th} process stream

δ = annual rate of return on the investment cost, Equation (1)

Subscripts

c = cold process stream

h = hot process stream

i, j = i^{th} and j^{th} stream or exchanger, respectively

s = steam

Superscript

$*$ = output value

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Thermal Cracking of Light Hydrocarbons and Their Mixtures

The kinetics and product distributions of the thermal cracking of binary and ternary mixtures of ethane, propane, *n*- and *i*-butane were determined in a pilot plant under conditions of residence time, temperature, total pressure, and dilution as close as possible to those prevailing in industrial operation. The kinetics and yields observed with ternary mixtures were compared with those obtained with binary mixtures and with pure components. The experimental selectivities were compared with those which would be obtained from separate cracking and subsequent addition of the product streams. The deviations between the two can be predicted by means of the so-called global kinetics selectivities, which are based upon the selectivities obtained from the pure components cracking and upon the global rates of cracking of the feed components in the mixture.

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SCOPE

The key feedstocks for the petrochemical industry are provided by the thermal cracking of hydrocarbons. The design of new plants and the analysis of existing units require basic information on the kinetics and on the product distribution obtained under varying operation conditions. So far, only fragmentary information is available. This paper focuses on the cracking of mixtures of light hydrocarbons in the C_1-C_4 range and particularly aims at providing the basis for deciding whether or not a gas mixture should be separated before cracking to optimize the yield pattern. Further, it investigates how far the cracking rates and the selectivities encountered in mixtures cracking can be predicted from information on the individual cracking of the feed components.

CONCLUSIONS AND SIGNIFICANCE

Accurate overall kinetic equations and product distributions have been obtained for the cracking of a number of light hydrocarbons and their binary and ternary mixtures. Ethylene and propylene yields from mixtures cracking are compared with those that would be obtained from separate cracking. The rigorous prediction of the selectivities or yields obtained from mixtures requires a set of rate equations accounting for the detailed radical reaction mechanism. So far this approach, which leads to serious computational problems, has not been applied to practical operating conditions.

The simple additivity rule (5), used until now in the literature, is not adequate for the prediction of the product distribution of mixtures cracking. It takes the selectivities obtained from the individual components at the desired mixture conversion. Obviously, these would have to be taken at the respective conversions of the feed components, which are generally not identical. These individual conversions can be predicted when the global kinetics of cracking of the components in the mixture are available. The selectivities predicted in this way are called in this paper selectivities based upon global kinetics, G.K.B. selectivities. Even these will not necessarily lead to a satisfactory prediction of the experimental values, since they do not completely account for the interaction between the reacting species. The following phenomenological rule, derived from the present work, may be of help in predicting the selectivities in a semiquantitative way: The selectivities obtained from mixtures cracking deviate from those based upon the additivity rule in the same direction as the G. K. B. selectivities.

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